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DISCUSSION OF ANALYTICAL METHOD OF DETERMINING THE LENGTH OF TRANSITION SPIRAL

(*Published in November, 1949*)

By T. F. Hickerson, Thomas R. Klingel, Carl F. Meyer, Joseph Barnett, J. J. Leeming, Allen G. Tyson, Donald Thompson, Ralph A. Moyer, and Michael V. Smirnoff

HIGHWAY DIVISION

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DISCUSSION

T. F. HICKERSON,[2] M. ASCE.—Dividing Eq. 4 by m,

$$a = \frac{v^2}{\rho} - g\,e \qquad (16)$$

which is the net acceleration that develops when rounding a superelevated curve. When $\rho = R$, Eq. 16 becomes

$$a_{max} = \frac{v^2}{R} - g\,e_{max} \qquad (17)$$

Dividing Eq. 17 by C yields the time, in seconds, in which this acceleration can take place without discomfort. Meanwhile the car has traveled a distance that is herein assumed equal to the length of the spiral S_s. Therefore,

$$t = t_1 - t_2 \qquad (18)$$

in which $t\left(=\frac{S_s}{v}\right)$ is the time for traveling at a constant speed the distance S_s; $t_1\left(=\frac{v^2}{R\,C}\right)$ is the time for acquiring the radial acceleration proportional to the centrifugal force; and $t_2\left(=\frac{g\,e_{max}}{C}\right)$ is the theoretical time allowed for rotating the car through an angle e_{max}.

Multiplying both sides of Eq. 18 by v, and restoring equivalent quantities,

$$S_s = \frac{v^3}{R\,C} - \frac{v\,g\,e_{max}}{C} \qquad (19)$$

which is the same as Eq. 12.

Using the values in Table 1—$D = 3°$; $R = 1{,}910$ ft; $v = 88$ ft per sec (60 miles per hr); $e_{max} = 0.07$; and $C = 1$—Eq. 18 gives: $1.81 = 4.06 - 2.25$, from which it is seen that the time for changing from the tangent to the fully superelevated curve is only 1.81 sec.

Eq. 19 shows that $S_s = 0$ if—

$$R = \frac{v^2}{g\,e_{max}} = \frac{V^2}{15\,e_{max}} \qquad (20)$$

—that is, no spiral is needed as far as the conditions of the formula are concerned. Even in such cases, however, no harm is done by the use of a spiral. In fact, the appearance and the ease of approach would be improved thereby.

NOTE.—This paper by Michael V. Smirnoff was published in November, 1949, *Proceedings*. The numbering of footnotes, equations, and illustrations in this Separate is a continuation of the consecutive numbering used in the original paper.

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The question now arises: Is Eq. 12 (or Eq. 19) a reliable formula? Certainly it is more comprehensive than the widely used formula:

$$S_s = \frac{v^3}{R\,C} \quad \text{or} \quad \frac{3.155\,V^3}{R\,C} \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which the effect of superelevation is omitted; however, both contain the factor C—which is assumed to be constant—but actually is quite variable. Thus, rearranging Eq. 12,

$$C = \frac{v^3}{R\,S_s} - \frac{v\,g\,e_{max}}{S_s} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which it is seen that C varies with speed, length of spiral, and the value of the superelevation ratio, e_{max}, reached on the curve, and with cross products of these functions, combined with the cube of the speed.

Recent actual tests on a number of curved roads in England by John Joseph Leeming and Archibald Niel Black[3] have shown "* * * that the comfort criterion is an approximate guide to the minimum radius to be used at any design speed, but not to the length of transition."

THOMAS R. KLINGEL,[4] ASSOC. M. ASCE.—The development of the spiral easement curve, and particularly the formulas for the determination of the length of curve, have been based on rule-of-thumb methods in many cases. The necessary refinement in highway design brought about by steadily increasing vehicle speeds has made it necessary to develop a more fundamental approach to many of the problems and to reanalyze the basic principles upon which many present-day formulas are based. Mr. Smirnoff takes a step in this direction, and it should encourage engineers to take a fresh look at the basic principles upon which many of their present design formulas are based.

When a car enters a curve, a centrifugal force is set up by the radial acceleration; and the superelevation sets up an opposing force which is the radial component of the force as a result of the acceleration of gravity. An ideal situation would exist if these two opposing forces were equal so that there would be no residual side force acting on the car. This situation is seldom realized since curves must ordinarily be designed for cars operating at all speeds from zero miles per hour to the maximum permissible speed on the highway. As a result, an unbalanced radial force will exist for all speeds except one. This force is given by Eq. 4.

The length of the easement curve as determined by Mr. Smirnoff is based on the premise that the increase in this force at maximum velocity must be at a uniform and gradual rate throughout the length of the spiral, so that the greater the unbalanced force, the longer the spiral, and the smaller the unbalanced force, the shorter the spiral. This leads to the entirely false conclusion that, if the speed and the superelevation are so adjusted as to eliminate the unbalanced force, no spiral is necessary. To illustrate, assume a 6° curve

[3] "Road Curvature and Superelevation: A Final Report on Experiments on Comfort and Driving Practice," by John Joseph Leeming and Archibald Niel Black, *Journal*, Institution of Municipal Engrs., February, 1950, p. 522.

[4] Associate Prof. of Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

(ρ = 955 ft) and a speed of 60 miles per hr (V = 88 ft per sec). The elevation required to balance this centrifugal force is 0.25 ft per ft of roadway width. With these values and Eq. 12 as derived by Mr. Smirnoff, the spiral length is found to be zero. This implies that the alinement can change from a tangent to a 6° curve and the superelevation from 0 to 0.25 ft per ft of width, instantaneously.

One of the primary functions of an easement curve is to provide for a gradual increase in the superelevation. The magnitude of the unbalanced force is determined entirely by the spread between the balancing velocity (V_B) and the maximum velocity (V_M), and is independent of the magnitude of the superelevation. It can be computed by the equation:

$$F = \frac{m}{\rho}(V^2_M - V^2_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The length of the spiral, as determined by Mr. Smirnoff, is directly proportional to the magnitude of the unbalanced force, and is, therefore, entirely independent of the magnitude of the superelevation. It is dependent only on the spread between the maximum velocity and the balancing velocity. The spread is entirely arbitrary in accordance with current design practice and, therefore, should not be the sole basis for determining the length of the curve, since it completely ignores the prime function of the spiral, to provide room within which to "run in" the superelevation.

Imagine a superexpress highway, in which the various traffic lanes are designed for definite increments of velocity. There might be a lane on a 6° curve on which all cars operate within the limits of a minimum velocity of 45 miles per hr, and a maximum velocity of 60 miles per hr. The balancing speed would probably be selected at 53 miles per hr so that the maximum unbalanced force would be 0.0553 lb per lb of weight in an outward direction at 60 miles per hr, and the same value in an inward direction at 45 miles per hr. The superelevation for this one lane would then be 0.197 ft per ft of width. The length of the easement curve for this lane, based on Eq. 12, with a value of C of 1.25 would be 125 ft. The same curve designed for speeds from 0 miles per hr to 60 miles per hr instead of from 45 miles per hr to 60 miles per hr would have a superelevation of only 0.10 ft per ft of width, and an easement curve 345 ft long, as taken from Table 1. In the first case, with a high average speed and a high superelevation, the spiral is only 125 ft long and the superelevation is "run in" at the rate of 0.158 ft per station along the spiral; in the second case, with a lower average speed and a lower superelevation, the spiral is 345 ft long and the superelevation is "run in" at the rate of 0.029 ft per station. These rates can be varied at will, depending only on how close the balancing velocity can be taken to the maximum velocity. In the first case, the spread between the balancing velocity and the maximum velocity is from 53 miles per hr to 60 miles per hr, and in the second case the spread is from 38 miles per hr to 60 miles per hr. This situation becomes more aggravated in designing traffic lanes for higher vehicle speeds.

The limitation of current practice in designing for all speeds on a single lane may keep the length of spirals, as designed by Eq. 12, within safe and comfortable limits since there is generally quite a spread between the balancing velocity and the maximum velocity. A theoretical approach to any problem, however, should provide for all the possibilities that may arise in future design problems, and if the formulas break down for other than current practice they are little better than rule-of-thumb methods.

It can be concluded that the length of easement curve is controlled by two independent factors: One is the maximum rate at which the superelevation can be "run in." This is the factor used in railroad design and for which the American Railway Engineering Association recommends a maximum rise in superelevation of $1\frac{1}{4}$ in. per sec. The second factor is the maximum rate at which the unbalanced force can be allowed to build up. This can be expressed as an increase in rate of radial acceleration per second, which is the value C in the paper, or as an increase in force in pounds per second per pound of weight of car. This latter value would be 0.031 lb per sec per pound of weight for $C = 1$ and 0.039 lb per sec per pound of weight for $C = 1.25$. In problems involving high superelevations with a relatively low spread between the maximum velocity and the balancing velocity, the first factor would apply; and in cases of relatively low superelevation and a high spread between the maximum velocity and the balancing velocity, the second factor would apply. It may be necessary to develop two separate and independent formulas for length of easement curve, one based on each of the foregoing factors. The length to be used in any case would be the larger value given by these two formulas. The use of a formula for the length of spiral with complete disregard for the magnitude of the superelevation or the rate at which it is to be "run in" can lead to some very erroneous conclusions, and it is probably equally true that the determination of the length of spiral, without regard for the rate at which the unbalanced force will build up, is equally false.

CARL F. MEYER,[5] M. ASCE.—To the best of the writer's knowledge Mr. Smirnoff has presented, for the first time in an American publication, a mathematical expression for the length of a transition spiral which includes the effect of superelevation.

Charles M. Noble,[6] M. ASCE, has pointed out that the use of superelevation cancels out part or all the centrifugal acceleration, depending on the speed of the vehicle. This effect was also discussed at length by Mr. Leeming.[7] In neither of these instances, however, was a mathematical relation given; yet in an earlier paper[8] Mr. Leeming presented a relation of substantially the same form as Eq. 15*b*. Although this equation appears to be theoretically correct, it produces peculiar results under some conditions. For

[5] Prof. of Civ. Eng., Worcester Polytechnic Inst., Worcester, Mass.

[6] "Thoughts on Highway Design Research As Related to Safety of Vehicle Operation," by Charles M. Noble, *Proceedings*, Highway Research Board, National Research Council, Vol. 17, 1937, p. 247.

[7] "The General Principles of Highway Transition Curve Design," by John Joseph Leeming, *Transactions*, ASCE, Vol. 113, 1948, p. 868.

[8] "Road Curvature and Superelevation," by John Joseph Leeming, *Road Paper No. 7*, Inst. C. E. October, 1942.

example, at "equilibrium speed" (that speed at which the centrifugal force is exactly balanced by the component of the weight of the vehicle toward the center of the superelevated curve) the value of e_{max} must be equal to $V^2/(15\,R)$. The length of transition required by Eq. 15*b* then becomes zero. This is an absurd result, yet it is required by the theory.

A simpler form of Eq. 15*b* for the usual case in which V is greater than the equilibrium value is found by substituting $15(e + f))$ for V^2/R, f being the familiar "side friction factor" or "cornering ratio." The result is

$$S_s = \frac{47\,V\,f}{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

A relation of this basic form was given by Elmer R. Haile, Jr.,[9] Assoc. M. ASCE, although he recommended that it should not be used because of the short spiral lengths produced when f is small. There is no necessity to discard the theoretically correct relation given by Eq. 15*b* or Eq. 24 solely for this reason. It is important, however, to consider the rotational effect. Although an extremely short spiral to a superelevated curve will result in no centrifugal sensation if traversed at equilibrium speed, yet the rotational change about the axis of the car may be too rapid for comfort or safety.

Long years of operating experience on spiraled superelevated railroad track have led the American Railway Engineering Association to recommend that minimum spiral lengths be based on attaining superelevation across standard gage track at a desirable maximum rate of $1\frac{1}{4}$ in. per sec. Highway and railroad operations are by no means analogous; but, in the absence of research on the motor vehicle rotational rate at which discomfort begins, there is logic in the tentative adoption of the same rate used successfully on railroads. (Both Mr. Haile and T. T. Wiley, M. ASCE, have advocated[9,10] consideration of the rotational effect.)

A rise of 1.25 in. per sec across the track gage of 4.71 ft is equivalent to about 0.022 ft per ft per sec. Consequently, $S_s = 1.47\,V(e/0.022)$, or

$$S_s = 67\,V\,e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The two formulas for minimum desirable spiral length are separately inconsistent for certain conditions. Eq. 24 gives values which are too short when f is small ($S_s = 0$ at equilibrium speed), whereas Eq. 25 gives values which are too short when e is small ($S_s = 0$ for nonsuperelevated curves). However, by using each formula within its proper range, the resulting minimum spiral lengths will produce neither unsafe values of f nor an uncomfortable rate of angular rotation.

The spiral length at which the change from one equation to the other occurs is found by equating the two values of S_s. Thus, $47\,V\,f/C$ governs the choice of minimum S_s when $e < 0.7\,f/C$; and $67\,V\,e$ governs S_s when $e > 0.7\,f/C$.

Rotational change, as represented by Eq. 25, governs minimum spiral lengths for practically all values of superelevation normally used on the open

[9] Discussion by Elmer R. Haile, Jr., of "The Modern Express Highway," by Charles M. Noble, *Transactions*, ASCE, Vol. 102, 1937, p. 1091.

[10] Discussion by T. T. Wiley of "The Modern Express Highway," by Charles M. Noble, *ibid.*, p. 1103.

highway. If the superelevation rate is less than one half the usual maximum for a curve of the given radius, Eq. 24 will usually govern.

The practical value of the rotational change method of choosing minimum spiral lengths is well illustrated in the case of the Pennsylvania Turnpike. The spirals on this highway were designed on the basis of a rotational change of about 0.02 ft per ft per sec—practically the value used in obtaining Eq. 25. The generally favorable operating characteristics of these spirals, as observed on high-speed tests,[11] are undoubtedly a result, in no small measure, of the constant (and comfortable) rate of rotational change experienced on successive curves.

Until recently, valid objection to this method of selecting spiral lengths might have been made on the ground that it does not necessarily produce a constant value of C. The assumption that C must be constant on a highway spiral (as it must, of necessity, be in the case of a spiral on railroad track) has never been conclusively demonstrated; it has been merely taken for granted. On the other hand, the careful field tests and statistical analyses made by Messrs. Leeming and Black[3,12] (which were described in part by Mr. Leeming in another paper[7]) show that C varies between such wide limits that there is no natural rate of turning the wheel, such as is represented by $C = 2$ or by any other constant value.

From Mr. Leeming's experiments it appears that safety and comfort depend mainly on the value of f, and that a driver slows down to reduce f, not to reduce C. Relatively large values of C (far in excess of 2 ft per sec^3) were frequently recorded as producing an "imperceptible" degree of discomfort, as long as the accompanying values of f were moderate (not greater than approximately 0.15).

In view of the foregoing research, it is felt that Mr. Smirnoff has placed too much emphasis on the term C, to the exclusion of the more important term f. The writer cannot agree with the author (as stated in his discussion of Eq. 12) that: "The entire question of the length of spiral thus reduces to a proper value of C * * *," nor that (as stated following Eq. 15*b*):

> "From previous discussions of the subject in engineering literature it seems reasonable to assume that the value of C in Eqs. 15 should be about 1 ft per sec^3."

Neither of these statements is borne out by the most extensive research made thus far—namely, that of Messrs. Leeming and Black.

Moreover, the author did not point out that Eqs. 15*a* and 15*b* do not necessarily yield the most desirable length of transition, but merely the minimum length required if an assumed rate of change of centrifugal acceleration (C) is not to be exceeded.

The paper seems to have as its purpose, proof that spirals may be unnecessarily long owing to neglect of the superelevation factor $v\,g\,e/C$. However, it

[11] "Curve Design and Tests on the Pennsylvania Turnpike," by Kenneth A. Stonex and Charles M. Noble, *Proceedings*, Highway Research Board, National Research Council, Vol. 20, 1940, p. 429.

[12] "Road Curvature and Superelevation: Experiments on Comfort and Driving Practice," by John Joseph Leeming and Archibald Niel Black, *Journal*, Institution of Municipal and County Engrs., December, 1944, p. 137.

should not be overlooked that practical considerations—such as suitable gutter grades, or pleasing appearance—may require even longer spirals than determined by Eq. 15*b* or Eq. 25.

Engineers interested in this problem should be indebted to Mr. Smirnoff for his careful mathematical analysis, and particularly for pointing out the fallacy (and its magnitude) of neglecting the effect of superelevation on the length of the transition curve.

JOSEPH BARNETT,[13] M. ASCE.—It is important to be reminded that superelevation should be provided for when using the rate of change of centripetal acceleration in computing the length of the transition curve. To be sure, the author counterbalances the reduction in the length of transition due to superelevation by suggesting a lower value for the constant than is generally used in highway practice; but there is no doubt that Mr. Smirnoff's approach is correct and to be preferred to the use of formulas that omit the effect of superelevation. The value of the constant should be determined experimentally, as the author suggests, but the writer feels that the effort has a low priority. There is no need for determining the length of transition precisely, and research in driver behavior and other aspects of highway engineering have a much higher priority.

Transitions should be used in all cases except on very flat highway curves. Length can vary over a wide range and any curve that varies in sharpness with the length is satisfactory. Mr. Smirnoff was wise not to advocate a particular shape of curve as has been done so often for the clothoid, cubic parabola, the A.R.E.A. (American Railway Engineering Association) curve in the United States, and for the lemniscate in England. It makes little practical difference how a transition curve is obtained or what exact curve is used.

A system of highways that has been admired for its appearance and smoothness of operation is the parkway system of Westchester County, New York; yet the transitions were obtained without using a mathematical curve. Instead, the line was laid down on topographic contour maps by the use of a spline, which is a flexible strip of wood or plastic held on the drawing board with weights. (The general location and alinement, of course, were first determined by reconnaissance in the field.) The stiffness of a spline causes the line to form transitioned curves naturally. The line was then computed as a sequence of compound curves, frequently three in number, the central curve being sharper than the outer ones, to fit the spline line best. When located in the field the line was considered satisfactory for grading purposes only. When forms were laid for paving, the fact that the line did not adequately conform to transitioned curves became evident; lining in by eye was the final step. Lateral shifts of the forms amounted to several inches and in many cases more than a foot. Considering that the computed line already was a form of transition, these shifts were appreciable. This method had the advantage of direct observation of the relation of horizontal alinement to vertical alinement, but the method was laborious. It is much more satisfactory to locate transitioned curves in the first place; and, with adequate tables, the

[13] Chf., Urban Road Branch, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

work of computation and location is no more laborious than that required for compound curves. The method of computing transitioned curves to fit spline lines has been used by the United States Bureau of Public Roads on all its park and forest roads, and the results are just as satisfactory, if not more satisfactory, than the method used in Westchester County.

Recognizing that precision in length of spiral is not necessary, the writer, in 1938, developed a set of tables[14,15] which makes it just as easy to design and locate curves with transitions as to design and locate simple curves, provided the design is made initially as transitioned curves and not as simple curves to be adjusted later for the addition of transitions.

J. J. LEEMING.[16]—In 1942 the writer proposed[8] a formula—

$$S_s = \frac{1}{C}\left[\frac{v^3}{R} - v\,g\,(e_{\max} \pm b)\right] \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

—which differs from Eq. 12 only by the addition of a symbol b to account for the camber. With traffic passing on the right, Eq. 26 applies to the left-hand curve when b is positive. In a strict sense, C should be Q, the rate of change of lateral force.

On being first introduced, much criticism was directed at Eq. 26 because, if camber is neglected, as in Eq. 12, the length of transition becomes zero at the "hands-off" speed. At first, it is difficult to account for the anomaly—although the difficulty is no greater than a similar one in the conventional formula—that the same length of curve is given no matter what the superelevation may be. Both of these anomalies were explained by experiments[12] in 1944, which showed that the rate of change of force is not a valid criterion for the design of transition curves. There is no such thing as a "safe" or "comfortable" rate of change of force so the equation is meaningless. In 1950, the writer and Mr. Black have proposed[3] an empirical method for the design of transition curves which includes an alinement chart for the solution of the rather complicated regression equation.

ALLEN G. TYSON,[17] M. ASCE.—An improvement in the equations for the length of the transition, developed independently by the writer, has the form:

$$S = \frac{3.15\,V^3}{C_o\,R}\,(1 - k) \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which k denotes the superelevation factor defined as the ratio of actual superelevation to the centrifugal ratio; and C_o is the allowable value of C on a roadway that is not superelevated, with $k = 0$. Since the practice of superelevating a curved pavement is more common than that of inserting transitions,

[14] "Transition Curves for Highways," by Joseph Barnett, Bureau of Public Roads, U. S. Dept. of Commerce, U. S. Govt. Printing Office, Washington, D. C., 1940.

[15] Discussion by Joseph Barnett of "Mathematical Examination of the Highway Transition Spiral," by John Oran Eichler and Howard W. Eves, *Transactions*, ASCE, Vol 111, 1946, pp. 978–979.

[16] County Surveyor, Dorsetshire, England.

[17] Asst. County Surveyor, Flintshire County Council, North Wales, Great Britain.

a more important relationship is

$$C_k = \frac{3.15\ V^3}{S\ R} = \frac{C_o}{1 - k} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Eqs. 27 and 28 show (in theory, at least) how transitions should be shortened as a higher superelevation factor is used; and they may also explain why the British practice, with a usual k-value of 0.33 to 0.40, tends toward a lower value of C_k than the American practice, with a k-value as great as 0.56. It is also interesting to recall that W. H. Shortt[18] states: "A curve which cannot be given superelevation is more in need of easing at the ends than one that can be superelevated." This statement clearly anticipates most subsequent conclusions, without putting them on a mathematical basis.

It is gratifying to the writer to read Mr. Smirnoff's opinion that the range of C (or rather, C_k)—namely, between 1.25 ft per sec^3 and 3 ft per sec^3 is small. In Great Britain this same range is considered by some highway engineers to be so unreasonably large that the use of C for design purposes should be abandoned entirely. One of these men has in fact expressed the opinion[19] that it is

> "* * * distressing to find the attitude that because a certain basis of design (i.e. the use of C) has been used in the past, its use should be perpetuated irrespective of the fact that evidence has been produced showing it to be fundamentally unsound."

The evidence referred to consists of admittedly extensive experiments, probably of the type which Mr. Smirnoff considers to have led to inconclusive results, and, as an unashamed reactionary, the writer shares Mr. Smirnoff's view that it should not be difficult to agree on some fairly conservative value for C_o to be used in design, say, in the range of from 0.5 ft per sec^3 to 1.00 ft per sec^3.

As long ago as 1945, the writer[20] suggested the following arbitrary sliding-scale values for C_o:

$$C_o = 1.05 - 0.0045\ V \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Substituting Eq. 29, with the British superelevation factor $k = 0.40$, in Eq. 28,

$$C_k = 1.75 - 0.0075\ V \ldots\ldots\ldots\ldots\ldots\ldots (30a)$$

and, with the value $k = 0.5625$ in use in the United States,

$$C_k = 2.40 - 0.01\ V \ldots\ldots\ldots\ldots\ldots\ldots (30b)$$

The object of the sliding scales is to shorten the spiral lengths on slow speed roads where the topography often limits the tangent lengths available and to lengthen the spirals for high design speeds which may well increase in the future.

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[18] "A Practical Method for the Improvement of Existing Railway-Curves," by W. H. Shortt, *Proceedings*, Inst. C. E., Vol. 176, 1909, pp. 97–99.

[19] *Roads and Road Construction*, March, 1950, p. 85.

[20] "The Design of Partly Transitional Curves," by A. G. Tyson, *Roads and Road Construction*, November, 1945, pp. 357–363.

DONALD THOMPSON.[21]—The formula for the length of a transition curve, as presented in this paper, is a real contribution to engineering literature. The formula is theoretically correct since it includes the effect of superelevation, and it should prove to be extremely useful not only in determining standards for transition curve design, but also, by its application to the investigation of existing transition curves, in arriving at a safe value for the maximum allowable rate of increase of the unbalanced centrifugal acceleration. The rate of increase of the unbalanced centrifugal acceleration, usually designated as C in transition curve literature (designated Q by Messrs. Leeming and Black[3]) has been the subject of much controversy among highway engineers. Values ranging from 1 to 3 have been used in the formula—

$$S = \frac{3.15\ V^3}{C\ R} \qquad (31)$$

—but, since Eq. 31 is incorrect because it ignores superelevation, the values of C experienced when traveling a transition curve did not correspond at all with the values assumed by the designer in calculating the length of the transition curve. With the formula given by Mr. Smirnoff, however, true values of C can be computed from tests of existing curves, and design standards can be made to conform more closely to actual driving practice.

In fact, present reports of actual driving experience on curves supply considerable information as to the comfortable and maximum values of C. Messrs. Leeming and Black, in reporting the results of many observations of driving practice on curves,[3] found that of the curves graded E (sensations of ateral acceleration imperceptible to observers) 77% had a C-value greater than 1.00, 36% had a C-value greater than 2.00, and 10% had a C-value greater than 4.00. As a result of the high speed tests on the Pennsylvania Turnpike,[11] Kenneth A. Stonex and Mr. Noble recommended an absolute maximum value of $C = 3.54$. These tests were run at speeds of 100 miles per hr and more, whereas the design speed of the Pennsylvania Turnpike for sections with radii larger than 1,432 ft was 70 miles per hr. The computed value of C for the turnpike curves at the design speed of 70 miles per hr is 1.20. In view of the foregoing tests, the author's recommendation for using a maximum value of $C = 1.00$ should certainly be adopted by highway engineers.

For those engineers who might have found the derivation of Mr. Smirnoff's formula unduly complicated, a more simple derivation, using the notation of the paper, but without the use of calculus, follows:

A vehicle traveling a superelevated curve at a speed greater than that for which the curve is superelevated experiences a centrifugal acceleration of

$$a = \left(\frac{v^2}{g\ R} - e\right) g \qquad (32)$$

Letting C equal the rate of increase of a, in feet per second cubed,

$$C = \frac{a}{t} \qquad (33)$$

[21] Civ. Engr., Bureau of Eng., Los Angeles, Calif.

in which t is the time, in seconds, required to traverse the transition curve. Also,

$$t = \frac{S}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

Therefore,

$$C = \frac{a}{S/v} = \left(\frac{\frac{v^2}{g\,R} - e}{S/v} \right) g = \frac{v^3}{R\,S} - \frac{e\,g\,v}{S} \quad \ldots\ldots\ldots\ldots\ldots (35)$$

Converting Eq. 35 to miles per hour by substituting 1.467 V for v,

$$C = \frac{47.2\,V}{S} \left(\frac{V^2}{15\,R} - e \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

and $S = \frac{3.15\,V^3}{R\,C} - \frac{47.2\,e\,V}{C}$, which is the same as Eq. 15*a*.

Current practice regarding the length of transition curves will not be greatly affected by the use of Eq. 15*a*. There are two methods in general use at the present time for calculating the length of a transition curve. The federal government standards for interregional highways[22] state that transition curves shall be of such length that the difference in slope between longitudinal profiles, separated by the width of one traffic lane, shall not be greater than 1 in 200. These standards also state that all curves sharper than 1° shall be superelevated and that the superelevation shall be such as to counterbalance completely the centrifugal force of a vehicle traveling at 0.75 of the design speed of the road, except that the superelevation shall not exceed 0.12 ft per ft under normal conditions and 0.08 ft per ft where snow or ice may be encountered. If traffic lanes are assumed to be 12 ft wide, the foregoing conditions regarding maximum slope and superelevation result in the following expression for the length of a transition curve:

$$S = \frac{200 \times 12 \times (0.75\,V)^2}{15\,R} = \frac{90\,V^2}{R} \quad \ldots\ldots\ldots\ldots\ldots (37)$$

Substituting Eq. 37 in Eq. 36 for S and also substituting $\frac{(0.75\,V)^2}{15\,R}$ for e in the same equation,

$$C = 0.0153\,V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

With this method of determining the length of a transition curve, C would vary from 0.765 at 50 miles per hr to 1.071 at 70 miles per hr.

The formula for the length of a transition curve which has had perhaps the widest use in highway design is

$$S = \frac{1.58\,V^3}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Substituting this expression in Eq. 36 and using the amount of superelevation recommended for interregional highways, $C = 0.875$. The use of Eq. 39 for transition curve lengths results in a constant value for C at all design speeds.

[22] "Interregional Highways," *House Document No. 379*, U. S. Govt. Printing Office, Washington, D. C., 1944.

Eqs. 37 and 39 give equal values for S at 57 miles per hr. At velocities greater than 57 miles per hr, Eq. 39 will give the greater value and should be used for design speeds of 60 miles per hr or greater. For velocities less than 57 miles per hr, Eq. 37 gives the greater length and should be used for design speeds from 30 miles per hr to 55 miles per hr. Under the foregoing conditions both Eq. 37 and Eq. 39, now in general use, will give values of C well below the maximum value of 1.00 recommended by the author, and their continued use is recommended by the writer.

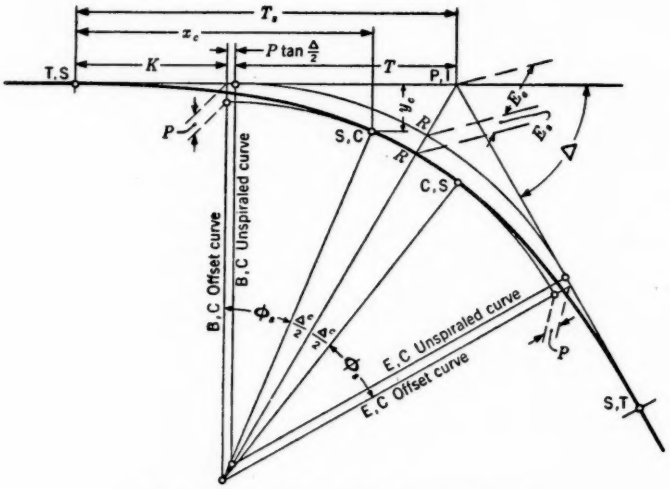

FIG. 1.—ELEMENTS OF THE TRANSITION CURVE

Since the tendency of the formula presented by Mr. Smirnoff is to permit shorter transition curves than were previously thought desirable, and since the shorter the transition curve, the less shift there will be in the path of a vehicle, it may be desirable to eliminate transition curves completely when the shift in the vehicle path is small enough to be taken up in one traffic lane. If transition curves can be eliminated without any sacrifice of safety, it would certainly be desirable, since there can be no question but that a spiraled alinement is more complicated to design and construct than one containing only circular curves. The interregional highway standards state that all curves with radii less than 2,865 ft shall be designed with transition curves. In the writer's opinion this rule is too general. The true criterion as to whether or not transition curves should be used is the amount the circular curve must be shifted to insert the transition curves. In Fig. 1 this shift is designated as P. Also,

$$\theta_s = \frac{S}{2R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

$$K = \frac{S}{2} - \frac{S^3}{240\,R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

$$y_c = \frac{S^2}{6\,R} - \frac{S^4}{336\,R^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and

$$x_c = S - \frac{S^3}{40\,R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The maximum shift in the path of a vehicle if it follows the spiraled alinement is

$$E_s = P\left(\sec\frac{\Delta}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Assuming that the maximum space available for maneuvering laterally in a 12-ft traffic lane is about 2.00 ft, and substituting this value for E_s in Eq. 44,

$$P = 2.00\left(\cos\frac{\Delta}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

For a 90° central angle, P would be 1.41 ft; that is, when P is less than 1.41 ft a transition curve would be unnecessary since there is sufficient width in the traffic lane for the vehicle to make its own transition. Therefore, it would seem to involve no compromise with safety to state that, when P is less than 1.00 ft, transition curves can be eliminated completely and the superelevation transition can be run out on the tangent and the circular curve.

It can be shown that

$$P = \frac{S^2}{24\,R} - \frac{S^4}{2{,}688\,R^3} + \frac{S^6}{506{,}880\,R^5} - \cdots\cdots\cdots\cdots\cdots (46)$$

The first term of Eq. 46 will give values accurate enough for determining whether or not transition curves should be used. When P is less than 1.00 ft, in the writer's opinion, radii do not require transition curves. Drivers will be able to create their own transitions, maintain sufficient side clearances between adjacent vehicles, and still remain in one traffic lane without the use of transition curves.

RALPH A. MOYER,[23] ASSOC. M. ASCE.—The author is to be commended for developing a term representing the effect of superelevation as a refinement of the widely used formula given in Eq. 13 for the length of the highway transition spiral. In 1934 the writer published a comprehensive report[24] in which the mathematical analysis for the formula in Eq. 13 was given, together with experimental data covering the various phases of vehicle operation on highway curves and their application to highway design. In that report the writer presented considerable data in support of the well-known fact that superelevation makes it easier to steer a vehicle on a curve, and that it makes the ride more comfortable. The data also indicated that the length of the transition spiral could be made shorter on a curve with superelevation than on a curve with the same radius but without superelevation. The writer, however, did not develop a term in the formula in Eq. 13 to express, on a mathematical basis, the shortening in the length of the spiral for various values of superelevation. This refinement has now been provided for in Eq. 15.

[23] Prof. of Civ. Eng. and Research Engr., Inst. of Transportation and Traffic Eng., Univ. of California Berkeley, Calif.

[24] "Skidding Characteristics of Automobile Tires on Roadway Surfaces and Their Relation to Highway Safety," by R. A. Moyer, *Bulletin No. 120*, Eng. Experiment Station, Iowa State College, Ames, Iowa, 1934.

The analysis by Mr. Smirnoff is made only on a force basis. From the discussion by the author and the data presented, one gains the impression that the transition spiral is required solely to define the curved path of the vehicle that will provide a constant rate of change of radial acceleration in terms of the unbalanced centrifugal force developed by the vehicle on the curve, and that, presumably, a transition spiral is not required for the purpose of developing the maximum superelevation on the curve.

According to Eq. 15*a*, the computed length of spiral, S_s, for a 6° curve (with $e_{max} = 0.10$, $C = 1$, and $V = 60$ miles per hr) is 177 ft, whereas the length required to develop the maximum superelevation as computed by the term $\frac{v\,g\,e_{max}}{C}$ is 236 ft. It is the writer's contention that the S_s-value applies only when it is greater than the $\left(\frac{v\,g\,e_{max}}{C}\right)$-value, and that the latter value should be used when it is greater than the S_s-value; that is, the transition spiral should always have a length equal to, or greater than, the length required to develop the maximum superelevation; and it is on this basis that the length should be great enough to permit the driver to steer the vehicle in such a way as to develop, at a safe and comfortable rate, the radial acceleration provided by (1) the unbalanced centrifugal force, and (2) the centripetal force obtained by the use of superelevation on the curve.

Although the writer recognizes the validity of the author's mathematical analysis on a force basis, the writer's study and experimentation on the subject of vehicle behavior on curves have led him to believe that a better understanding of the transition curve length requirements can be obtained by an examination of the steering requirements on curves, with and without superelevation, rather than by using only a mathematical analysis on a force basis.

The principal concern of the driver and the highway designer is that the driver should be able to steer the car around the curve safely at a reasonable speed consistent with the speed design standard for the given highway. The driver is not concerned and has little knowledge of the centrifugal, frictional, and superelevation forces involved in steering the car around the curve. He should only be concerned with the control and movement of the steering wheel to keep the car in the desired curved path.

The actual forces at the rim of the steering wheel within the normal driving speeds rarely exceed 5 lb and usually are of such a small magnitude that they are difficult to measure. The centrifugal and frictional forces that may be developed by the vehicle on the curve, as the result of turning the steering wheel, may exceed 1,000 lb; and it is evident, therefore, that the driver with little effort can set into motion very large forces that can be controlled only by the skilful manipulation of the steering wheel. It is for this reason that highway curves should be designed to permit a gradual change when the vehicle is being steered from the tangent to the curve, where large forces are required to hold the car on the given curved path at high speeds.

In Fig. 2 the average variations in the turning or steering angles at the steering wheel and the resulting coefficients of friction are shown for three popular make passenger cars driven at various uniform speeds on 5° curves and

10° curves, with and without superelevation. The turning angles at the steering wheel increase for each curve as the speed is increased to some power approaching the square of the speed. At slow speeds on the superelevated curves, negative steering angles are required to steer the car along the desired path of the curve since, at these speeds, the superelevation will pull the car toward the center of the curve. At a speed of 30 miles per hr on both superelevated curves, the gravity force provided by the superelevation is equal to the centrifugal

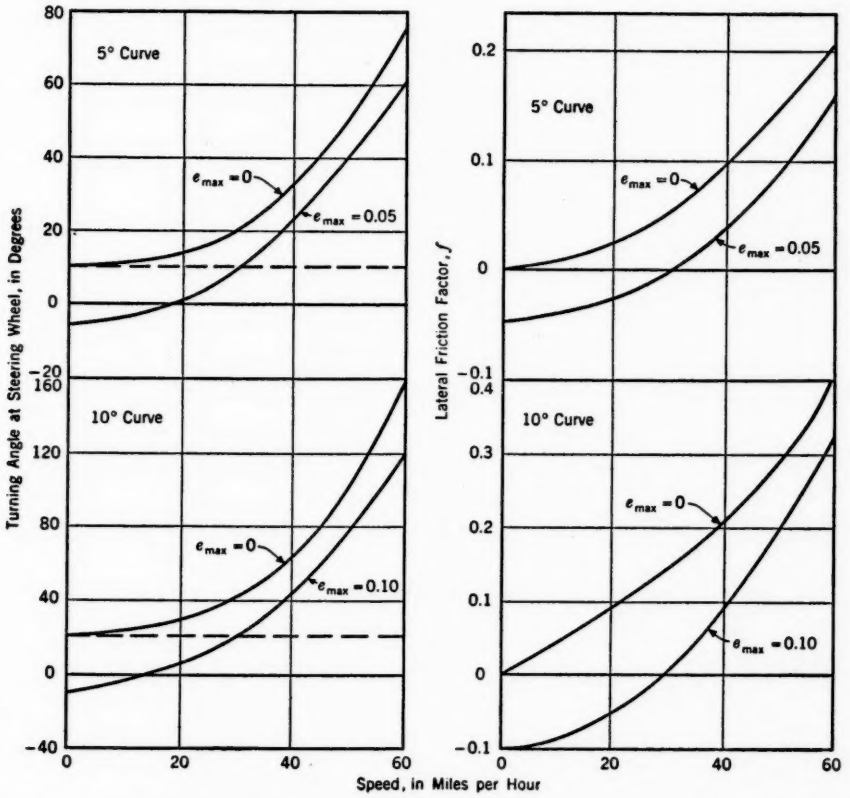

FIG. 2.—VARIATIONS IN THE TURNING ANGLE AT THE STEERING WHEEL AND THE LATERAL FRICTION FACTOR WHEN DRIVING PASSENGER CAR AT VARIOUS UNIFORM SPEEDS ON 5° AND 10° CURVES WITH AND WITHOUT SUPERELEVATION

force and the steering angles required are the same as the wheel base angles. When measured at the front wheels, the steering angle is the angular deviation of the front wheels from the car axis; and, at the speed where the lateral forces acting on the car are balanced, it is equal to the angle subtended by the wheel base of the car on the given curve.

For speeds greater than 30 miles per hr on these curves, the steering angles increase rather sharply since the tires at both the front and rear must be turned toward the center of the curve by increasing amounts to develop the slip angles

required to produce the tire friction and road friction which are necessary to hold the car in the desired curved path at the given speed.

Actually, at 60 miles per hr on the 5° curve, the steering angle (amounting to about 60°) should be considered as an easy turn. The spiral curve length for this curve, using Eq. 15 (with the recommended value of $C = 1$ and a design speed of 60 miles per hr), is 451 ft. Neglecting superelevation, and using $C = 2$, the corresponding spiral length according to the widely used Eq. 13 is 296 ft. Tests conducted by the writer on a two-lane concrete pavement (with traffic lanes 9 ft wide and using path markers to define the path of the car) revealed that the car could be held in a path within 1 ft of the center of the traffic lane despite the fact that the pavement was constructed as a circular curve without spirals, and that a tangent runoff section, only 80 ft long, was used to develop the full superelevation at the point of curvature (PC) and point of tangency (PT) of the circular curve.

In the writer's judgment, the approach to this 5° curve, and the steering on the curve, could have been improved considerably if the curve had been laid out with transition spirals at both ends. The 450-ft spiral length recommended by the author seems unnecessarily long. A person who regularly drives at 60 miles per hr on a tangent would have no difficulty in turning through a 60° angle at the steering wheel in 450 ft or in the 5.1 sec required to travel this distance at 60 miles per hr. In fact, the tests on this curve indicated that the transition could easily be made in one half of that distance (225 ft), which corresponds to a C-value of 2 ft per sec^3.

Actual tests were not run on a 5° curve without superelevation; but the results of many tests on curves with varying amounts of superelevation provided the basis for the data in Fig. 2 for the 5° curves and the 10° curves without superelevation. It is logical that the centripetal force on the superelevated 5° curve provided by the superelevation should be replaced by a frictional force on the 5° curve without superelevation. This frictional force is obtained by a fairly uniform increase in the slip angles and steering angles for each speed as shown for each curve. On the 5° curve with $e_{max} = 0.05$, the average increase in the steering angles should amount to about 10° except in the range of speed from 50 miles per hr to 60 miles per hr where the increase will probably reach a maximum of 15°. On the 10° curve with $e_{max} = 0.10$, the average increase should amount to 20° and may reach a maximum of 40° at 60 miles per hr. This increase in the steering angle is a measure of the increased length of transition required on a curve without superelevation as compared to the length required at the same speed on a curve of the same radius with superelevation. Thus, with a 25% increase in the steering angle at 60 miles per hr for the 5° curve, the length of the transition curve should be 25% longer for the curve without superelevation than for the curve with a superelevation of $e_{max} = 0.05$.

If the value of $C = 1$ is used for both terms in the formula in Eq. 15*a*, the S_s-value for the 5° curve ($V = 60$ miles per hr) is 451 ft when $e_{max} = 0.05$, and 593 ft when $e_{max} = 0$; that is, the increase in spiral curve length, according to the formula, is about 32% as compared with a 25% increase based on the steering angle requirements. The analysis of the length requirements of a spiral curve, in terms of steering angles, indicates that higher C-values may be used in

computing the length requirements for developing superelevation than for developing the radial acceleration produced by the unbalanced centrifugal force. However, the writer believes that the difference in the C-values and in the length requirements obtained by this method of analysis is not large enough to require the use of two C-values in Eq. 15*a*. The author's mathematical analysis indicates that the same C-value should be used for both terms in Eq. 15*a*. At least that seems to be his interpretation inasmuch as he uses only one C-value in Eq. 15*b*.

Although the author did not discuss the length requirements of transition curves to develop the maximum superelevation, the factors that make this an important consideration in computing the length of the transition spiral should be emphasized as follows: Superelevation is the most important factor that railroad engineers have to consider in determining the length of transition for railroad curves because, in railroad practice, a large part of the centrifugal force on a curve is balanced by a gravity force provided by superelevation. It is unsafe to depend on a large amount of friction or flange pressure force to hold the trains on a curved track. Since superelevation is the dominant factor in the design of railroad curves, the length of the railroad spiral has generally been determined by using a uniform rate of rotational change in the cross slope of the track to develop the maximum superelevation. The maximum rate of rotation used in determining the length of railroad spirals has been established by the American Railway Engineering Association (A.R.E.A.) as the rate at which the superelevation may be attained without discomfort to passengers. A rate of $1\frac{1}{4}$ in. per sec, which is equivalent to a rotational change of 2% of cross slope per sec at the assumed speed, is recommended by the A.R.E.A.[25]

The C-value corresponding to a rotational change of 2% of cross slope per sec is equal to 32.2 (0.02) or 0.644 ft per sec^3. This is a very low value for C; and, although it may be a desirable value for providing a comfortable train ride, it is far below the values used or required in highway practice. It has been almost a universal highway practice in the United States to lay out the center line using circular curves without spirals and to develop the maximum superelevation, in part or entirely, on the tangent. Obviously, the lengths of the transition sections used to develop the maximum superelevation have been short. The writer is familiar with the curve standards in Iowa, and it is on these curves that practically all his test driving was done. The Iowa standard for developing superelevation for many years called for an 80-ft transition section on the tangent ahead of the PC and following the PT for all curves on which $e_{max} = 0.05$ or less; and a 125-ft transition section was used when $e_{max} = 0.062$ or more. Although the Iowa standard did not indicate a speed factor or time value, the application of a suitable speed value (as, for example, 60 miles per hr on a 5° curve and 45 miles per hr on a 10° curve, for which test data are given in Fig. 2) will yield C-values using the author's formula, $\frac{v\,g\,e_{max}}{C}$, of 1.76 ft per sec^3 and 1.70 ft per sec^3, respectively, for the 5° curves and the 10° curves. The rotational change on these curves was three times greater than the usual

[25] *Bulletin No. 108*, A.R.E.A., February, 1911.

railway standard, but the higher rate was neither hazardous nor uncomfortable in the tests at the accepted design speeds for these curves.

In the writer's opinion, the requirements for obtaining a comfortable ride in a railroad train are quite different from those for an automobile on a paved highway; and the needs are likewise different. The shock absorbing rubber tires of the automobile are a major factor contributing to a comfortable ride; and, at the same time, the passengers in the automobile do not require the smooth ride demanded in a railroad dining car. It is not uncommon on a highway to have a change in pavement crown of 2 in. in 30 ft; whereas, according to railway curve standards, at least 80 ft would be required to make such a change. On the basis of these tests and observations, the writer recommends $C = 2$ as the maximum value for design purposes and $C = 1.5$ as a desirable standard using Eq. 15.

Transition spirals have been used on some notable projects, such as the Pennsylvania Turnpike and the new highways in national parks and national forests, with gratifying success. The writer has observed widespread reluctance on the part of state highway departments, however, to adopt transition spirals as standard practice in the design of new highways, or in making adequate provision on existing highways to permit the driver of a vehicle to make the transition from the tangent to the circular curve with the same factor of safety and comfort as is possible on curves designed with transition spirals. There is little doubt that in the construction of more than 90% of the rural highways in the United States since the 1920's, the horizontal alinement has consisted of tangents and circular curves without spirals. Many design engineers have insisted that transition spirals are not necessary because they have observed that, since drivers are able to steer all types of vehicles with reasonable safety within the 10-ft traffic lanes on old paved highways built without transition spirals, they surely should be able to do so on the wider lanes on new highways. The new formula developed by the author strengthens this belief, since it introduces a term which reduces the length of spiral when compared with the length computed according to the formula now in general use.

The writer concurs with the general belief that transition spirals are not required up to a certain point, but he has a suspicion that few engineers actually know the range in curvature and speed within which it is resonably safe to design curves without spirals and beyond which the spiral curve requirements increase at an alarming rate. Reference to Fig. 3 will clarify this important factor in spiral curve design.[25a] The P-value is an important one in an analysis to determine the spiral curve length requirement, since it is a measure of the amount by which drivers tend to "cut in" toward the center of a curve when making the transition from the tangent to the circular curve (see curves A and B, Fig. 3). Actually the highway is usually laid out with a center-line alinement consisting of a tangent and a circular curve, such as curve A; but the natural path of the vehicle (which both the theory and tests indicate is the easiest to follow) is that shown by curve B, Fig. 3. If P is only 0.14 ft, it should not be

[25a] For further clarification, the writer has prepared tabular spiral curve data, including various values of C, f, and P, at design speeds of 70 miles per hr and 50 miles per hr. A copy of these data will be sent on request to the writer from the Institute of Transportation and Traffic Eng., Univ. of California, Berkeley 4, Calif.

necessary to use transition spirals to hold the vehicle within the traffic lane; but if P is 16.15 ft, the vehicle will be forced outside the traffic lane unless transition spirals are built into the pavement, or unless an exceptionally skilful driver is at the wheel developing large C-values. As a practical consideration, on all curves and at all speeds where the P-value is less than 1.0 ft, the driver should have no difficulty holding the vehicle within a 12-ft, or even a 10-ft, traffic lane on highways built with unspiraled circular curves. Thus, under this specification (for a design speed of 70 miles per hr and using the writer's recommended value of $C = 1.5$) transition spirals should not be necessary on 2° curves or less; and, if a value of $C = 2.0$ is accepted (as tests indicate that it can be), spirals should not be necessary on 3° curves or less in so far as lane width requirements are concerned.

Although transition spirals are not necessary on 3° curves or less at 70 miles per hr, as far as the lane width requirements are concerned, they may be necessary to meet the superelevation requirements. Thus, the length of spiral

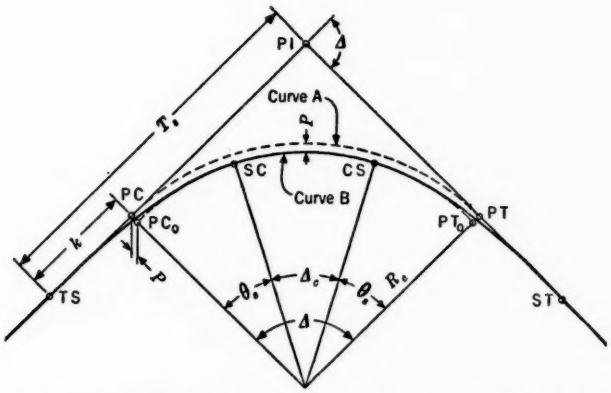

FIG. 3.—CIRCULAR CURVE WITH SPIRAL TRANSITIONS AT BOTH ENDS

required to effect the rotational change that will develop the maximum superelevation of 0.10 ft per ft, based on the new term proposed by the author $\left(\frac{v\,g\,e_{max}}{C}\right)$, and using $C = 2.0$, is 165 ft; and, using $C = 1.5$, the length is 220 ft. Even the superelevation requirement can be met on an unspiraled circular curve if one half of the required length to develop superelevation is on the tangent curve and the other half is on the circular curve. The superelevation should be started on the tangent at a point corresponding to the tangent to spiral point (TS) in spiral curve design; it should then be increased uniformly over the computed lengths of 165 ft or 220 ft, to a point on the circular curve corresponding to the curve to spiral point (CS) where the maximum superelevation is reached. A pavement built in the foregoing manner, where the P-value is less than 1.0 ft, would permit identically the same operation with equal comfort on curves without transition spirals as on curves with transition spirals. However, on 4° curves, the P-values range from 1.82 ft to 4.65 ft, and transition spirals

from 250 ft to 400 ft in length are required for the 70-mile speed. On 6° curves the P-values range from 6.97 ft to 10.88 ft, and spiral lengths of from 400 ft to 500 ft are required.[25a]

At a speed of 70 miles per hr, on a 6° curve, with $e_{max} = 0.10$, the writer has found[25a] the lateral friction factor, f, to be 0.24; and, with $e_{max} = 0.06$, $f = 0.28$. The recommended maximum friction value for a design speed of 70 miles per hr is $f = 0.10$. Therefore, a 6° curve should not be considered acceptable on a highway built for a design speed of 70 miles per hr. However, on the Pennsylvania Turnpike it was found necessary to include 6° curves and vehicles are frequently operated at 70 miles per hr on these curves. These curves were designed with spiral lengths of 320 ft and 360 ft which Mr. Noble, special highway engineer for the Pennsylvania Turnpike, reported as a subnormal length.[26] Tests on the 6° curves as reported by Messrs. Stonex and Noble were run at speeds up to an average uniform value of 85 miles per hr which is well above the usual maximum 60-mile design speed for 6° curves. It is significant that in these tests a maximum value of $C = 3.54$ was measured and reported by Messrs. Stonex and Noble on the very same basis as the C-value according to the formula developed by the author. Although the drivers in these tests were highly skilled, and the C-values are close to the maximum that may be developed when entering a curve, these test results indicate that the author's recommended value of $C = 1$ is too low and that the writer's recommended values of $C = 1.5$ and $C = 2.0$ provide an ample margin of safety. The measured and calculated lateral friction factor, f, in these tests averaged 0.3869 and 0.3955, respectively. This is well above the f-value considered safe and was so reported by Messrs. Stonex and Noble. The most positive evidence of the steering hazard at 85 miles per hr on the 6° curve was the marked deviation in the paths of the test cars with deviations of 12 ft and 16 ft measured over the 24-ft paved width available. These deviations in the paths of the test cars are an indication of the P-values to be expected when driving at high speed on a 6° curve where the drivers follow spiral curve paths in making steering adjustments at various points along the curve very similar to those required in going from the tangent to the circular curve.

The spiral curve data compiled by the writer[25a] indicate that, on the sharper curves permitted for a speed of 50 miles per hr, larger values of P are encountered for the corresponding C-values and f-values than in the curve design at 70 miles per hr. At 50 miles per hr the accepted maximum lateral friction factor (f) in curve design is 0.16, and a 9° curve is the recommended maximum curve using $e_{max} = 0.10$. For a 9° curve the P-value is 4.09 ft when $C = 1.5$ and 2.62 ft when $C = 2.0$. These P-values are so large that many drivers will find it impossible to maintain control of the vehicles within 10-ft, or even 12-ft, traffic lanes if the curves are designed as circular curves without spirals. The data for speeds of 50 miles per hr indicate that on a 9° curve spiral lengths of from 200 ft to 300 ft are required. The corresponding data for the 6° curve show that a P-value of 0.98 ft is required. Therefore, transitional spirals are not necessary on 6° curves or less, designed for 50 miles per hr, for the same reasons as those

[26] "Curve Design and Tests on the Pennsylvania Turnpike," by Kenneth A. Stonex and Charles M. Noble, *Proceedings*, Highway Research Board, National Research Council, Vol. 20, 1940, p. 440.

given previously in the discussion of the 2° curves and 3° curves. However, provision should be made in the design of all superelevated curves without spirals to introduce the superelevation in about the same way that it would be introduced if transition spirals were used.

The use of the author's formulas for computing the length of transition spirals requires the acceptance of the spiral as a curve of uniformly changing curvature and a constant rate of change of radial acceleration. The argument has frequently been advanced in discussions of transition sprials that drivers approach curves along many different paths and that it is impossible to design a curved path which all drivers will follow. It is further contended that different makes of vehicles have different steering characteristics which influence the driver's control of the vehicle on curves.

The writer is fully aware of the many variations in the paths of cars when entering a curve due to the driver and the car; but it is his contention that control of the car is greatly simplified if the curves are designed with adequate superelevation and with transition spirals where needed, using a substantial factor of safety of about 3 at moderate speeds and 4 at high speeds. With a factor of safety of 3 or 4, the driver has considerable latitude as to the path he can follow and still stay within the 10-ft to 12-ft traffic lanes. At speeds below the design value, steering is futher simplified and the factor of safety increased. It is when drivers operate at speeds greater than the design speed that the unsafe steering characteristics of certain vehicles will be evident and exceptional skill on the part of the driver may be required to control the car and keep it within the traffic lane. The writer, therefore, favors the spiral curve path with its constant rate of change of curvature as the easiest and least hazardous path along which the average driver can steer a vehicle.

Messrs. Stonex and Noble[27] report results which show that in the Pennsylvania Turnpike tests the friction values (f) increased uniformly from zero at the TS of the test curve to a maximum value at the beginning of the circular curve. This supports the validity of the spiral curve theory in which the rate of change of radial acceleration is constant.

The author has made an important contribution in the development of a new term in the formula for determining the length of the transition spiral. This discussion has been offered in an attempt to clarify the author's analysis and its application to highway design in the hope that reasonable standards for the use of transition spirals will be developed and adopted by all highway departments.

MICHAEL V. SMIRNOFF[28] ASSOC. M. ASCE.—With the very interesting discussions presented on the question of transition spirals, it will be possible to make summaries and draw certain conclusions. The discussions show that the question of the length of transition curves has been approached from many different angles which are just as important as the unbalanced centrifugal force used in the derivation of Eq. 12 by the writer.

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[27] "Curve Design and Tests on the Pennsylvania Turnpike," by Kenneth A. Stonex and Charles M. Noble, *Proceedings*, Highway Research Board, National Research Council, Vol. 20, 1940, p. 443.

[28] Lecturer, Dept. of Eng., Univ. of California, Los Angeles, Calif.

The questions raised were: Whether the rate of change of radial acceleration, C, should be considered by highway designers as being constant; and how the length of transition spiral may be affected by such factors as the lateral friction factor, f, the rotational effect of the automobiles, the superelevation runout, the value of p on spirals, and the steering and the slip angles developed by automobiles on curves.

The Rate of Change of Radial Acceleration.—The important factor in Eq. 14 is the rate of change of the unbalanced centrifugal force which is certain to affect the drivers. Obviously there must be a definite limit of this rate of change of force that could be considered safe and comfortable by all drivers and riders. It is difficult to arrive at such a limiting value merely because its magnitude is much more a matter of personal reaction and judgment than the subject of an exact mathematical analysis; but, once a certain value of C is agreed on, there seems to be no reason to suppose that it should vary under different design conditions. The value of C may be variable as far as the rate of change of any acceleration may be variable; but, to a highway designer, it is no more variable than the speed which he considers constant for the particular highway he is designing.

Mr. Tyson presents arbitrary sliding scales for C in Eqs. 29 and 30 to make, as he points out, the spirals longer for higher speeds and to provide for the possible future increase in speeds. Such provisions are scarcely necessary, since the length of spiral, as shown by Eq. 12, is directly proportional to the speed and, thus, will be longer for higher speeds, whereas a possible future increase in speed will affect not only the length of transition spiral but also the circular curves, the superelevation, and other factors which will not be safe for higher speeds.

In regard to the proper value of C in Eq. 12, the question might well be asked as to how such a value as 1.5 ft per sec^3 will affect a man standing in a bus while the latter is moving along a transition curve, say, in 5 sec. If the unbalanced centrifugal force is the controlling factor in the design and the man weighs about 200 lb, $\frac{dF}{dt} = 9.3$ lb per sec. Thus, the man will be pushed by a gradually increasing force which will accumulate to an additional 9.3 lb at the end of each second, until finally the man will feel a thrust of 46.5 lb on the circular curve. To give this man 3 sec or 5 sec to brace himself against that force may spell the difference between an accident and escape, and it certainly will be more comfortable for him to brace himself in 5 sec than in 3 sec. The writer finds it impossible to support Mr. Leeming's contention that there is no such thing as a safe or a comfortable rate of change of force.

A word of caution should be offered about Mr. Thompson's suggestion to substitute Eq. 39 in Eq. 36 in order to determine the value of C. Eq. 39 was originally derived from Eq. 13 by using $C = 2$. Thus it will be possible to substitute only Eq. 13 in Eq. 36—giving a relation between C_{12} as used in Eqs. 36 or 12 and C_{13} as used in Eq. 13. The relation is

$$C_{12} = \left(1 - \frac{15\,R\,e}{V^2}\right) C_{13} \qquad (47)$$

For a specific example of a 6° curve, with $V = 60$ miles per hr and $e = 0.10$, $C_{12} = 0.6\ C_{13}$. Although this equation does not hold for other conditions, it gives a good idea of the corresponding values of C as used in two equations. Mr. Moyer states that he considers $C_{12} = 1.5$ as a desirable value, whereas Mr. Barnett[14] recommends $C_{13} = 2.0$, which corresponds approximately to $C = 1.2$. The writer is inclined to take the average of 1.2 and 1.5 which will give $C = 1.35$ for Eq. 12 or Eq. 15, as discussed subsequently.

The Lateral Friction Factor.—The desirable maximum value of the lateral friction factor, f, has been well investigated by Messrs. Moyer, Stonex and Noble, Barnett, and others. As reported by Mr. Moyer and D. S. Berry,[29] Assoc. M. ASCE, most states use $f = 0.15$ to determine a safe driving speed on curves for posting. To quote Messrs. Moyer and Berry,[30]

> "The general acceptance of this value is rather surprising because it is after all an arbitrary value at which the driver of a car senses some discomfort and where the hazard of skidding off the curve becomes apparent."

The authors also express the opinion that, for speeds greater than 60 miles per hr, a lower value should be used. Messrs. Stonex and Noble[26] came to the conclusion that $f = 0.10$ should be used for speeds of 70 miles per hr and greater.

The lateral friction factor enters a well-known equation,

$$e + f = \frac{V^2}{15\,R} \qquad (48)$$

For a given design speed, Eq. 48 gives two parameters from which a third unknown may be determined. This equation can be best employed to determine the maximum radius of a circular curve for a maximum rate of superelevation, e, and a desirable value of f or, else, to determine on a given curve the rate of superelevation for a desirable value of f.

Substitution of e from Eq. 48 in Eq. 15*a* gives Eq. 24, $S_s = \frac{47.2\ V f}{C}$, cited by Mr. Meyer. The value of superelevation, which may be found from Eq. 48, is usually rounded to the nearest one sixteenth of an inch; and, to find the final value of f for Eq. 24, Eq. 48 must be used. Eq. 24 is no more applicable than Eq. 15*a*, since either one can be used with the same results.

In case of recommendations for interregional highways, cited by Mr. Thompson, the balanced superelevation on curves would be designed for 75% of design speed, provided that e does not exceed 0.12. Thus, $f = 0$ when $0.75\ V$ is used instead of the V in Eq. 48. The proper substitution gives $e = \frac{0.5625\ V^2}{15\,R}$, and

$$\frac{e}{e+f} = 0.5625 \qquad (49)$$

[29] "Marking Highway Curves with Safe Speed Indications," by R. A. Moyer and D. S. Berry, *Proceedings*, Highway Research Board, National Research Council, Vol. 20, 1940, p. 399.

[30] *Ibid.*, p. 409.

Eq. 49 gives a definite relation between e and f, or $f = 0.778\ e$. When this value is substituted in Eq. 24, it becomes

$$S_s = \frac{36.7\ V\ e}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

Eq. 50, like Eq. 15*a*, is based on an unbalanced centrifugal force. Unfortunately, it will have no practical application, since, as it will be evident from further discussion, the unbalanced force will not be a controlling factor in the length of transition spiral when the superelevation is designed for 75% of design speed, and Eq. 50 will not hold for any other conditions.

It is interesting to note that Mr. Tyson gives $k = 0.40$ as the value for $\frac{e}{e+f}$ in Eq. 49, which is used in Great Britain. This value would make the superelevation of roads in Great Britain designed for about 63% of the design speed. Under these conditions, the unbalanced centrifugal force will be always a controlling factor in the length of transition spiral.

The Rotational Effect of Automobiles.—It was brought out by Messrs. Klingel, Meyer, and Moyer that the American Railway Engineering Association recommends a rise of $1\frac{1}{4}$ in. per sec for cars on superelevation runouts for a 4-ft $8\frac{1}{2}$-in. gage of track—which is equivalent to a rate of rotation of 0.0221 ft per ft per sec. In railroad practice the main consideration in the rotational effect of the cars is attached to the fact that on superelevation runouts the rail gradients remain unparallel, which results in one diagonal pair of wheels of a car truck having an uneven bearing pressure as compared with the other pair. This fact leads to an uneven compression of the springs, to the danger of the leading wheel climbing the rail, and to other complications which are absent in case of an automobile. Besides, automobile chassis are more flexible than are railroad car trucks and the rate of rotation for automobiles can be considerably greater than that for the railroad cars before any discomfort will be noticed by the riders.

If the rate of rotation of automobiles is designated by ϕ feet per foot per second and if any length of spiral S is considered to have a uniform rate of gradient $\frac{de}{dS}$, the time that an automobile will travel this length S at a speed of v feet per second will be S/v seconds. Thus, $\frac{S}{v}\phi = S\frac{de}{dS}$, or $\phi\ dS = v\ de$, and for the entire spiral $\phi \int_0^{S_s} dS = v \int_0^{e_{max}} de$, which gives

$$S_s = v\frac{e_{max}}{\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

Eq. 51, which reduces to Mr. Meyer's Eq. 25 when $\phi = 0.0221$, shows that the length of spiral needed on the basis of rotational effect of cars is the same function of v and e_{max} as the second term in Eq. 12—that is, $\frac{v\ g\ e_{max}}{C}$. If K in-

stead of C is used as a constant of proportionality and the two terms are equated,

$$K = g\,\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

and Eq. 51 becomes

$$S_s = \frac{v\,g\,e_{max}}{K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

In order to obtain a dimension for S_s, in feet, K must be expressed in feet per second.[3]

The interregional standards, cited by Mr. Thompson for the minimum length of transition spiral, to be such as not to exceed the difference in longitudinal profiles of 1 ft in 200 ft in one traffic lane are the requirements that limit the rate of rotation of cars to a certain maximum. Substituting in Eq. 51, $S_s = 200$ and $e = \frac{1}{w}$ (in which w is the width of one traffic lane) and solving for ϕ, $\phi = 0.00733\frac{V}{w}$ (in which V is in miles per hour). The section of interregional standards cited applies for speeds greater than 50 miles per hr, which means that, according to its provisions, the rate of rotation will be satisfactory even at high speeds. Thus, if $V = 70$ miles per hr is used on, say, an 11-ft pavement, $\phi = 0.0467$. This value is slightly more than twice the rate of rotation recommended on railroads—that is, 0.0442. If it is assumed then that $\phi = 0.0442$ is within the limits set by interregional standard requirements, $K = 1.42$ from Eq. 52.

The desirable value for K can also be verified by the standards of various highway departments. For instance, if those cited by Mr. Moyer for Iowa are used, a transition section of 125 ft is required for an e_{max} of 0.062 or greater, and one of 80 ft for an e_{max} of 0.05 or less. Substituting these values in Eq. 53 for a speed of 60 miles per hr, or 88 ft per sec, one obtains $K = 1.40$ or greater in the first case, and $K = 1.77$ or less in the second case. Similar values for K may be computed by using the standards of other highway departments. The foregoing examples show that $K = 1.35$ is within the limits provided by the existing highway standards.

The close correlation which obviously exists between the K-values in Eq. 53 and the recommended values for C in Eq. 12 really has a deeper significance. The second term in Eq. 12 represents the reduction, due to superelevation, in the length of transition spiral based on centrifugal force without superelevation. In such a case, the second term itself should be adequate for the lengths of spiral due to superelevation. It develops, then, that K in Eq. 53 and C in the second term of Eq. 12 are one and the same constant—which result gives a new basis for judging the proper value of C in Eq. 12. When the second term in Eq. 12 is greater than the length obtained by the entire equation, that term should be used for the length of transition curves. Mr. Moyer, no doubt, came to the same conclusion when he made his statement to this same effect. Thus, instead of using more or less arbitrary distances for superelevation runouts, either on a spiral or on a tangent, a more consistent expression of the second

term of Eq. 12 can be used for such purposes. It will provide better designed highways.

The Effect of p-Values on Spirals.—Mr. Moyer presents valuable experimental data regarding p-values. According to these data, drivers have no difficulties in negotiating a circular curve—that is, keeping in lane—without a transition spiral if the latter has a p-value equal to 1 ft or less. Mr. Thompson supports this contention by mathematical analysis.

On curves flatter than 2.5°, p-values greater than 1 ft are possible only at speeds greater than 70 miles per hr; and, on curves flatter than 5.5°, p-values greater then 1 ft are possible only at speeds greater than 40 miles per hr. Thus, within these speed limits, the degrees of curvature between 2° 30′ and 5° 30′ will offer a basis to judge such minimum lengths of transition curves beyond which the curves will apparently serve no useful purpose.

Using the first term in the series given by Eq. 46, substituting $5{,}730/D$ for R, and solving for the length of spiral, one obtains, $S = 370\sqrt{p/D}$. For $p = 1$ and $D = 2° 30'$ and 5° 30′, S is equal to 234 ft and 158 ft, respectively. Thus, relying on the presented data regarding p-values, it may be concluded that, from the standpoint of keeping a car in lane, the spiral will not be necessary on any circular curve if the computed length of spiral is 150 ft or less.

It has been also stated in discussions that on such curves a spiral may be necessary for the superelevation runout. The superelevation runout will be a controlling factor in the design of spirals when twice the value of the second term in Eq. 15*a* is greater than the first term. Setting such inequality and substituting e from Eq. 48, one can easily verify the fact that the superelevation runout will be the controlling factor when $e > f$. Such conditions may occur on relatively flat curves with high speeds or on sharper curves with lower speeds. In either case it would appear adequate to run the superelevation out on the tangent. For instance, the question may arise as to whether it would be necessary to introduce a spiral on a 3° curve designed for 60 miles per hr with $e = 0.07$. With $C = 1.35$ in Eq. 15*a*, $S_s = 263 - 147$, which shows that a 150-ft spiral is required for the superelevation runout. Although the design would be better if such a spiral were introduced, there could not be much objection to running the superelevation out on a tangent on such a curve. Thus, a minimum limit of 150 ft for spirals may serve the purpose in either case.

The Effect of Steering and Slip Angles on Spirals.—The steering and slip angles have been thoroughly investigated by Messrs. Moyer and Stonex and Noble.[26] Mr. Moyer presents an excellent discussion of the question and emphasizes that the C-value in Eq. 15*a* can be greater than 1 ft per sec^3 because of such requirements. Using his analysis, one can check the previously suggested value of $C = 1.35$ to meet the requirements for steering angles. On a 5° curve with $V = 60$ miles per hr and with $e = 0.05$ (which data are used by Mr. Moyer for illustration) the length of spiral will be $440 - 105 = 335$ ft, and 440 ft when $e = 0$—an increase of 31% as compared with the 32% estimated by Mr. Moyer for $C = 1$. The increase based on the steering angle requirement is about 25% from Fig. 2. It appears that the C-values do not appreciably affect the percentage increase in spiral lengths because of omission

of superelevation, and that with either C-value the length of spiral will fall within the limits needed for the increase in steering angles.

It is worthwhile to mention one additional useful purpose served by transition curves, which is not only to provide the highways with such alinement that will make it easy to keep the cars in lane, but also to eliminate the tendency for cars to leave a lane. A spiral that is too short will not eliminate a "so-called" tendency to "cut the corners," whereas a spiral that is too long unnecessarily increases the amount of field and office work and, thus, adds to the cost of construction.[31]

Conclusions.—From the paper and its discussions it can be concluded that:

1. Although individual opinion of engineers may differ somewhat regarding the proper value of C in Eq. 15*a*, the discussions indicate that if should not deviate appreciably from $C = 1.35$ ft per sec.[3] With this value, Eq. 15*a* becomes

$$S_s = 2.33\frac{V^3}{R} - 35\ V\ e_{max} \quad \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

2. When the second term in Eq. 54 is greater than the length computed by the entire equation, the value of that term should be used for the length of transition curve.

3. The length of transition curve will be the same in the foregoing two cases when the superelevation is computed for 70.7% of design speed. Such a balanced condition occurs when $e = f$.

4. If the required length of a transition curve is less than 150 ft, no serious defect in design will result in omitting the spiral.

5. The second term in Eq. 54 can also be used to determine the length of superelevation runouts on tangents when the spiral is omitted.

6. A better design in any case will result in attaining the maximum superelevation used on curves at the point of connection with a circular curve.

Many thanks are due to those who contributed their discussions and useful experimental data to clarify the purposes and requirements for certain lengths of transition curves. These ends have been ably served.

[31] "Transition Curves on Transportation Routes," by M. V. Smirnoff, thesis presented to the Dept. of Eng., Univ. of California, at Los Angeles, in January, 1950, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

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